Algorithms for Estimating Trends in Global Temperature Volatility

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Climate change

The scientific consensus is that

1. World-wide climate is changing.
2. This change is mostly driven by human behavior.

~~Global warming~~ ⟶ climate change: the distribution of temperature (and precipitation) is changing

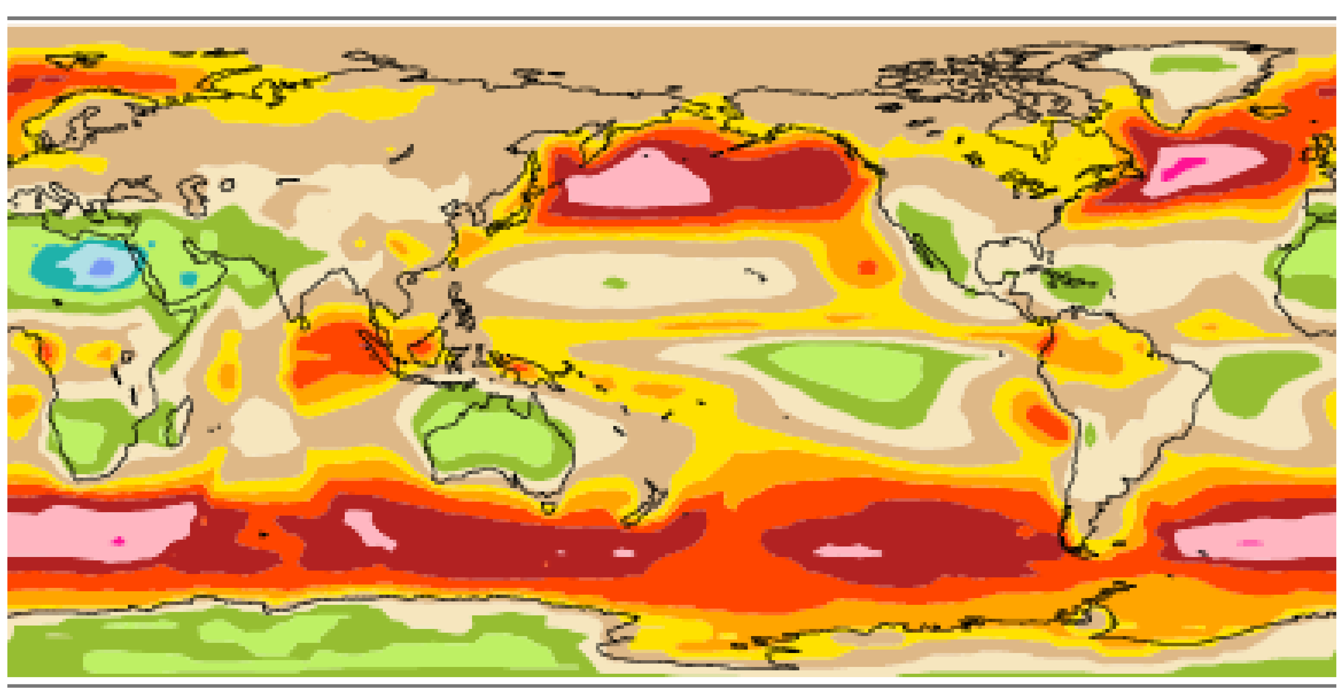
Increasing mean temperature understates the costs:

1. More frequent extremes have severe effects
2. Local discrepancies lead to more storms
3. Temporal dependencies mean persistence

Using weather satellites

Drivers of climate variation:

1. Ocean currents
2. Jet stream
3. Annular modes + El Niño/La Niña
4. Cloudiness



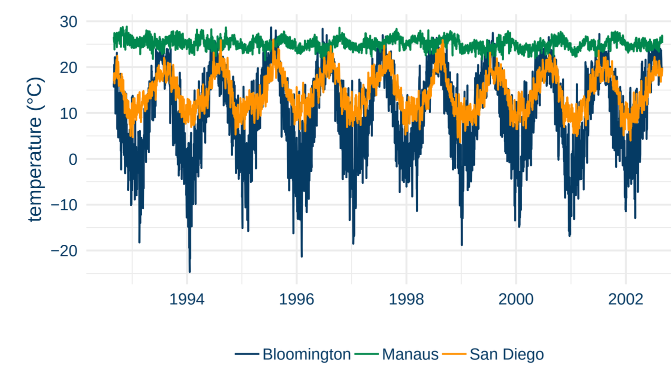
Source: NCAR CCSM3 Diagnostic Plots.

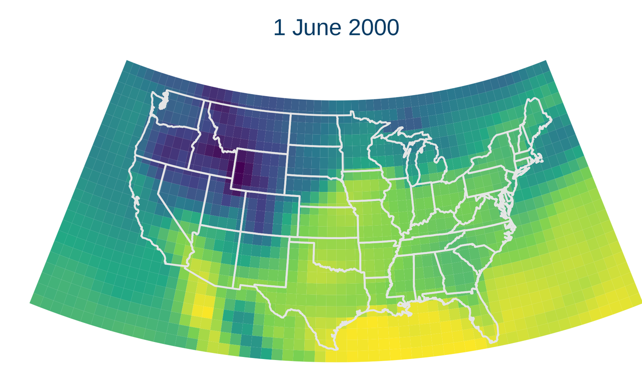
CLARREO satellite: monitor cloud top temperature vis-à-vis climate.

- Has yet to launch, no sooner than 2022
- Defunded in most recent federal budget

Satellite data

- 52,000 time series
- daily records over ∼ 40 years
- "trends" are local, nonlinear, not sinusoidal





Model: Trend in variance

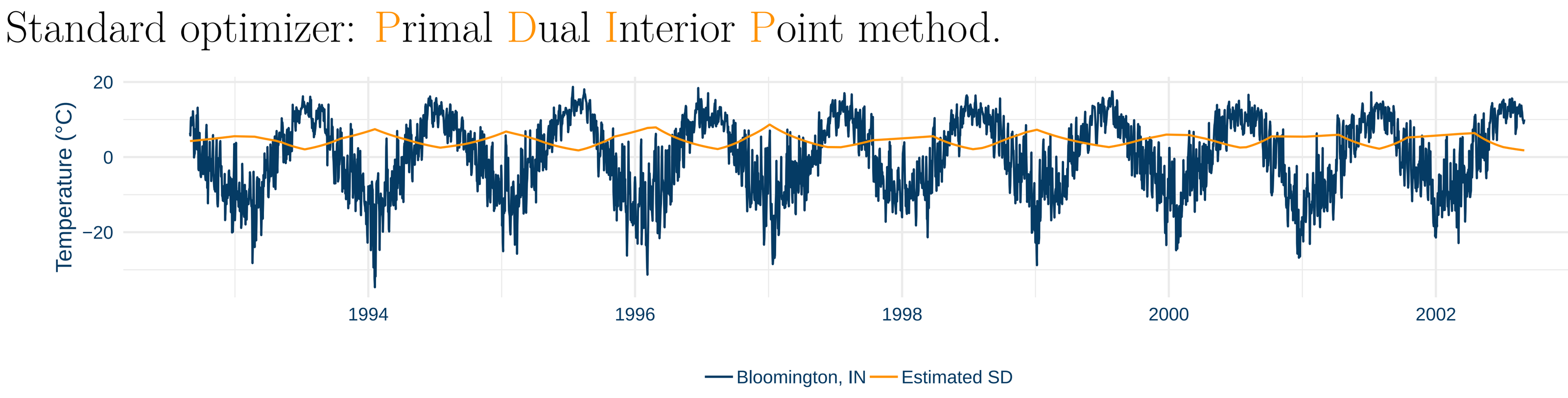
- y_{ts}: observed temperature at time t and location s.

$$y_{ts} \sim \mathrm{N}(0, \exp(h_{ts}))$$

- Estimate h, but it should be "smooth" relative to space and time.
- Use a matrix D + penalty to encode this smoothness.

$$\min_h \sum h_{st} + y_{st}^2 e^{-h_{st}} + \lambda \|Dh\|_1$$

Standard optimizer: Primal Dual Interior Point method.



Generic PDIP

1. Start with a guess $h^{(1)}$
2. Solve a linear system $[A(h^{(i)})u^{(i)} = v^{(i)}]$
3. Calculate a step size
4. Iterate 2 & 3 until convergence

$A^{(i)}$ changes each iteration, dense, and roughly $10^9 \times 10^9$.

Algorithms

For large memory, consensus ADMM

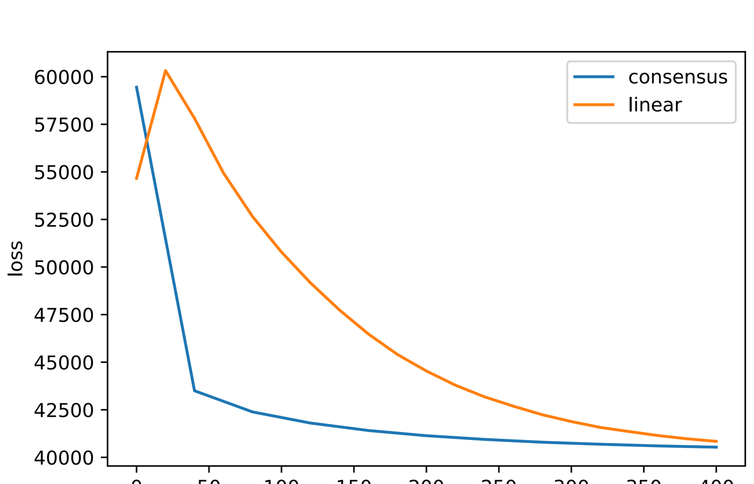
Input: data y, penalty matrix D, $\epsilon, \rho, \lambda_t, \lambda_s > 0$.
Set: $h \leftarrow 0$, $z \leftarrow 0$, $u \leftarrow 0$. ▷ Initialization
repeat
  $x_i \leftarrow \underset{x_i}{\mathrm{argmin}} - l(y_i \mid x_i) + \Lambda_{(i)}^\top |D_{(i)} x_i| + (u_i)^\top x_i + (\rho/2)\|x_i - \tilde{h}_i\|_2^2$.
  ▷ Update local vars using PDIP
  $h_k \leftarrow (1/S_k) \sum_{G(i,j)=k} (x_i)_j$. ▷ Global update.
  $u_i \leftarrow u_i + \rho(x_i - \tilde{h}_i)$. ▷ Dual update
until $\max\left\{\|h^{m+1} - h^m\|,\ \|h^m - x^m\|\right\} < \epsilon$
Return: h.

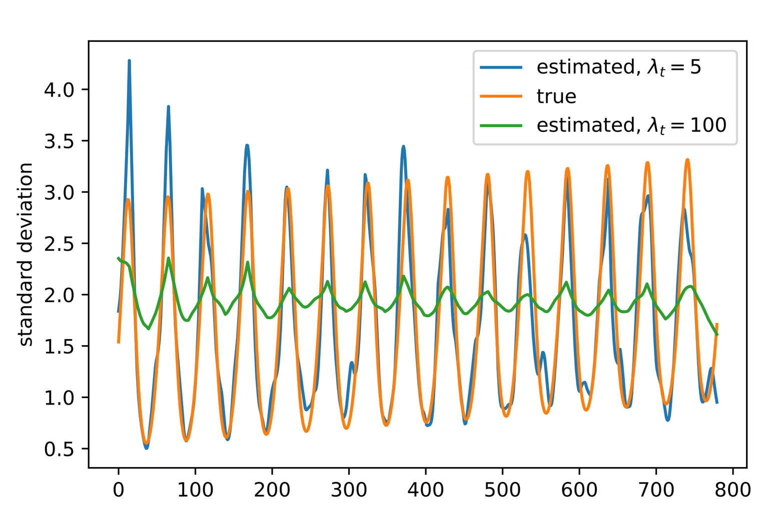
Linearized ADMM

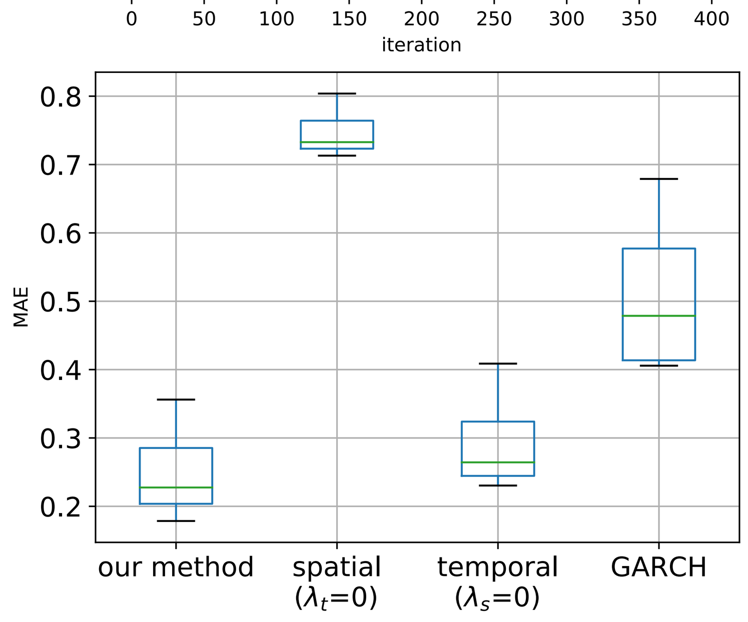
Input: data y, penalty matrix D, $\epsilon, \rho, \lambda_t, \lambda_s > 0$.
Set: $h \leftarrow 0$, $z \leftarrow 0$, $u \leftarrow 0$. ▷ Initialization
repeat
  $h_k \leftarrow \mathscr{W}\left(\frac{y_k^2}{\mu}\exp\left(\frac{1-\mu u_k}{\mu}\right)\right) + \frac{1-\mu u_k}{\mu}$ for all $k = 1, \ldots TS$. ▷ Primal update
  $z \leftarrow S_{\rho\lambda}(u)$. ▷ Elementwise soft thresholding
  $u \leftarrow u - z$. ▷ Dual update
until $\max\{\|Dh - z\|,\ \|z^{m+1} - z^m\|\} < \epsilon$
Return: z.

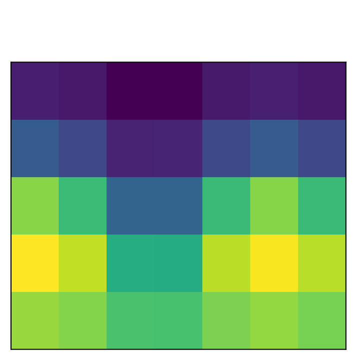
Simulations

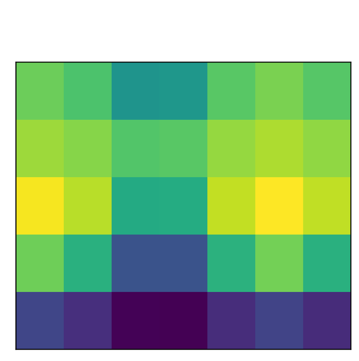
$$\sigma^2(t, r, c) = \sum_{k=1}^{K} W_k(t) \cdot \exp\left(\frac{(r - r_k)^2 + (c - c_k)^2}{2\sigma_k^2}\right)$$
$$W_k(t) = \alpha_k \cdot t + \exp(\sin(2\pi\omega_k t + \phi_k)).$$



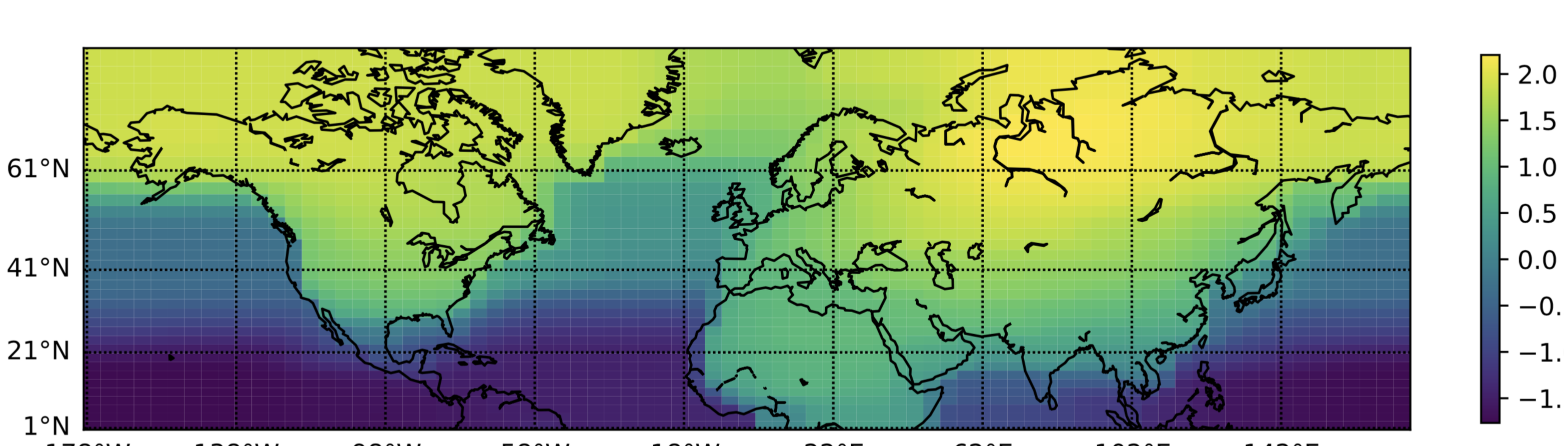


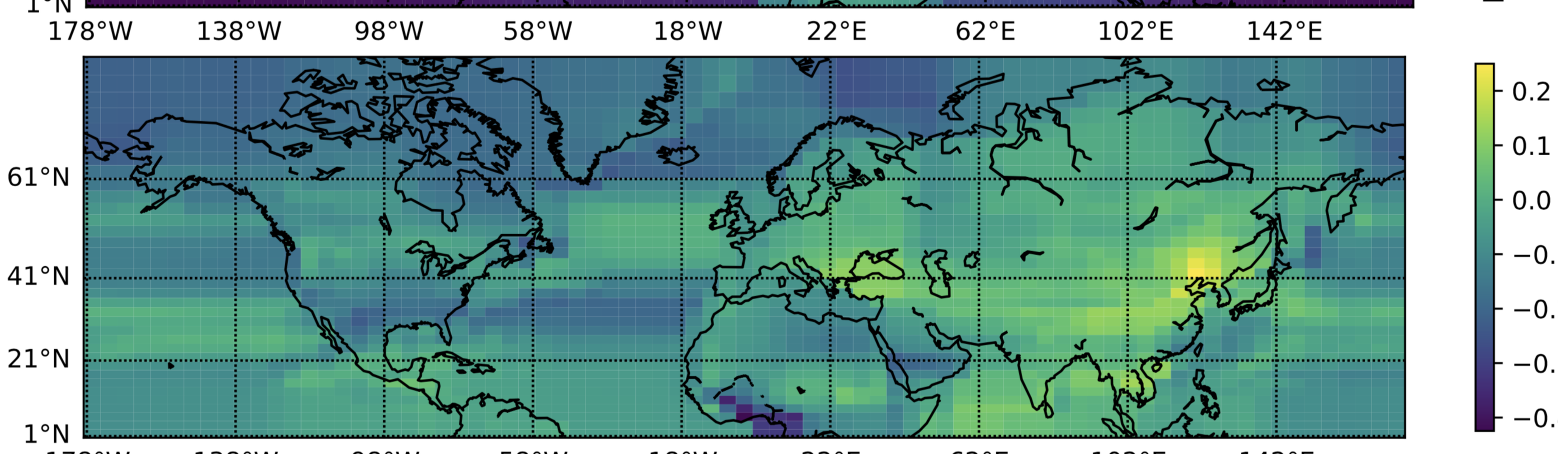






Data results





Acknowledgements

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Paper: Code:

My www: